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PROPAGATION OF DISCONTINUITIES IN HETEROGENEOUS ANISOTROPIC PLATES

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A. S. D. Wang and D. L. Tuckmantel

Mechanics and Structures Advanced Study Group
Research Report 72-7

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PROPAGATION OF DISCONTINUITIES IN HETEROGENEOUS ANISOTROPIC PLATES

By

A. S. D. Wang[1] and D. L. Tuckmantel[2]

ABSTRACT

Elastic stress waves propagating in thin, laminated composite plates are analyzed on the basis of a lamination theory. The theory is based on the Kirchhoff assumptions, but it includes the effects of shear deformation and rotary inertia, similar to Mindlin's theory for homogeneous isotropic plates. The individual layers comprising the plate are assumed to possess different thicknesses and material properties. In particular, each layer may be arbitrarily anisotropic. Thus, a general coupling in shear, bending, twisting and extensional effects is present in the plate constitutive relations. This coupling results in simultaneously coupled stress waves propagating in the plane of the plate. Several numerical examples involving laminated fiber-reinforced composite plates are presented.

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I. INTRODUCTION

The dynamics of heterogeneous anisotropic solids has been an important field of study in recent years. Increasing popularity in the use of modern composites as a structural material has necessitated intensive research into their material characterization. Their intrinsic properties are being investigated from both a phenomenological and microscopic point of view. This effort has resulted in the development of various lamination theories describing laminated anisotropic plate structures. The papers by Yang, Norris and Stavsky [1], and Whitney and Pagano [2] are among the most notable works. The former concerns plates made of layers possessing arbitrary anisotropy; the latter, with some modifications*, is valid for plates made of monoclinic layers, i.e., each layer of the plate possesses a mid-plane material symmetry. Both of these theories follow the basic approach set forth by Mindlin [4] for homogeneous isotropic plates, and include transverse shear deformation and the effect of rotary inertia. Recently, Chou and Carleone [5] improved the assumptions concerning the transverse shear deformation.

Such lamination theories, however, describe the macro-characteristics of the plate rather than the micro-characteristics that are effected by the inhomogeneous nature of the composite. Generally speaking, the macro-theory is accurate for plates subjected to static loading, but is deemed inaccurate when applied to stress wave problems. This is true in so far as the wave lengths are short compared with, say, the thickness of the material layers of the plate. But for low frequency waves with length longer than, say, the plate's thickness, the macro-theory may be regarded as a valid basis of analysis. Based on this assumption, Moon [6] recently investigated stress waves in a specially laminated fiber-reinforced composite plate, using the effective modulus theory which uncouples the transverse, bending and extensional displacements. A wave surface approach was used to describe the propagation of plane acceleration waves. This method yields the wave velocity surfaces in the plane of the plate.

---

*A comparison of the two theories is contained in Ref. [3].

The present paper is concerned with stress waves in laminates that are composed of layers possessing arbitrary anisotropy. General shear, bending, twisting and extensional coupling is present in the plate constitutive relations, resulting in simultaneously coupled wave surfaces in the plane of the plate. We follow general laminated plate theory and apply a control volume approach for the analysis [7]. Explicit solutions for the coupled wave surfaces and their velocities are obtained. Several numerical problems involving laminated fiber-reinforced composite plates are presented and their unique features discussed.

II. ANALYSIS

Let us consider a thin laminated plate of thickness h, Figure 1. The laminae comprising the plate are assumed to be individually homogeneous and anisotropic. Thus the in-homogeneity of the plate occurs only in the thickness direction. The constitutive relations for any one of the laminae are given by

$$\sigma_i = C_{ij}\, e_j \qquad i,j = 1,2,3,4,5,6. \tag{1}$$

where the C_{ij}'s are the elements of the stiffness matrix, the stresses, σ_i, are defined as $\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$, $\sigma_3 = \sigma_z$, $\sigma_4 = \sigma_{yz}$, $\sigma_5 = \sigma_{zx}$, $\sigma_6 = \sigma_{xy}$, and the strains, e_j, are defined in the same manner as the stresses.

Following the theory developed by Yang, Norris and Stavsky [1], we assume the displacement field,

$$\begin{aligned} u &= u^o(x,y,t) + z\,\psi_x(x,y,t) \\ v &= v^o(x,y,t) + z\,\psi_y(x,y,t) \\ w &= w^o(x,y,t) \end{aligned} \tag{2}$$

where the coordinate system (x,y,z) is shown in Figure 1 and u, v and w are the displacements in the x,y and z directions, respectively, u^o, v^o and w^o are the displacement components at z = 0, in the x,y and z directions, respectively and ψ_x and ψ_y are rotations about the y

and x axis, respectively.

The stress and moment resultants are related to the displacements by (c.f. Equation 14, Ref. [1]).

$$\begin{Bmatrix} N_x \\ N_y \\ Q_y \\ Q_x \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} & A_{15} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{24} & A_{25} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{14} & A_{24} & A_{44} & A_{45} & A_{46} & B_{14} & B_{24} & B_{46} \\ A_{15} & A_{25} & A_{45} & A_{55} & A_{56} & B_{15} & B_{25} & B_{56} \\ A_{16} & A_{26} & A_{46} & A_{56} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{14} & B_{15} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{24} & B_{25} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{46} & B_{56} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} u,_x^o \\ v,_y^o \\ w,_y^o + \psi_y \\ w,_x^o + \psi_x \\ u,_y^o + v,_x^o \\ \psi_{x,x} \\ \psi_{y,y} \\ \psi_{x,y} + \psi_{y,x} \end{Bmatrix} \tag{3}$$

where

$$(N_x, N_y, Q_y, Q_x, N_{xy}) = \int_{-h/2}^{+h/2} (\sigma_1, \sigma_2, \sigma_4, \sigma_5, \sigma_6)\,dz$$

$$(M_x, M_y, M_{xy}) = \int_{-h/2}^{+h/2} (\sigma_1, \sigma_2, \sigma_6)\, z\,dz \tag{4}$$

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{+h/2} C_{ij}\,(1, z, z^2)\,dz , \quad i,j = 1,2,4,5,6 \tag{5}$$

and the notation $\psi_{x,y}$, e.g., represents partial differentiation of ψ_x with respect to y.

We now consider a wave which originates at an arbitrary point in the plate, for convenience let us say at the origin of the (x,y,z) system, and propagates in the x,y plane. At any given instant, the wave surface is denoted by S, as shown in Figure 1. Let $\tilde{n}$ be the normal of S at a point, A, on S, and let $\tilde{s}$ be the tangent of S at

the same point. The wave surface S is assumed to propagate in the direction $\tilde{n}$ with a constant speed c.

Let $\bar{u}$,* $\bar{v}$, $\bar{w}$, $\bar{u}^o$, $\bar{v}^o$, $\bar{w}^o$, ψ_n and ψ_s be the displacements (see Equations (2)) referred to the local coordinates (n,s,z). The kinematic (continuity) conditions across S at any given point on S require (see [7]),

$$[\bar{u}^o] = [\bar{v}^o] = [\bar{w}^o] = [\psi_n] = [\psi_s] = 0$$

$$[\bar{u}_{,n}^o,\ \bar{v}_{,n}^o,\ \bar{w}_{,n}^o,\ \psi_{n,n},\ \psi_{s,n}] = \frac{1}{c}\,[\bar{u}_{,t}^o,\ \bar{v}_{,t}^o,\ \bar{w}_{,t}^o,\ \psi_{n,t},\ \psi_{s,t}] \tag{6}$$

and $(\bar{u},\ \bar{v},\ \bar{w},\ \psi_n,\ \psi_s)_{,s} = 0$

where [ ] represent a discontinuity of the enclosed quantity across S.

With these conditions, the plate constitutive relations, Equations (3), when transformed to the local coordinates (n,s,z) yield**

$$\begin{Bmatrix} N_n \\ N_{ns} \\ M_n \\ M_{ns} \\ Q_n \end{Bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{16} & \bar{B}_{11} & \bar{B}_{16} & \bar{A}_{15} \\ \bar{A}_{16} & \bar{A}_{66} & \bar{B}_{16} & \bar{B}_{66} & \bar{A}_{56} \\ \bar{B}_{11} & \bar{B}_{16} & \bar{D}_{11} & \bar{D}_{16} & \bar{B}_{15} \\ \bar{B}_{16} & \bar{B}_{66} & \bar{D}_{16} & \bar{D}_{66} & \bar{B}_{56} \\ \bar{A}_{15} & \bar{A}_{56} & \bar{B}_{15} & \bar{B}_{56} & \bar{A}_{55} \end{bmatrix} \begin{Bmatrix} \bar{u}_{,n}^o \\ \bar{v}_{,n}^o \\ \psi_{n,n} \\ \psi_{s,n} \\ \bar{w}_{,n}^o \end{Bmatrix} \tag{7}$$

To establish the dynamic relations across the wave surface, let us define a control volume which is located on S at point A,

*Quantities with a bar on top are referred to the (n,s,z) coordinate system.

**For the transformation of A_{ij}, B_{ij} and D_{ij} from the (x,y,z) system to the (n,s,z) system, refer to Ref. [8].

as shown in Figure 1. Since the control volume moves with the wave front, an observer fixed with it sees a normal influx of mass entering with a speed U_1, and a normal efflux of mass leaving with a speed U_2,

$$
\begin{aligned}
U_1 &= c - \bar{u}_{1,t} \\
U_2 &= c - \bar{u}_{2,t}
\end{aligned} \tag{8}
$$

where the subscripts 1 and 2 refer to the properties in front of and behind the wave front, respectively. Thus the steady-state conservation of mass for the control volume yields,

$$\int_{-h/2}^{+h/2} (\rho_2 U_2 - \rho_1 U_1) dz = 0 \tag{9}$$

where ρ is the mass density of the material, U is the particle velocity relative to the wave front and the subscripts 1 and 2 refer to properties in front of and behind the wave, respectively.

It is noted that condition (9) is satisfied by a more restrictive condition, resulting from the classical thin plate assumptions, namely;

$$\rho_2 U_2 = \rho_1 U_1 \tag{10}$$

The force and moment resultants acting upon the control volume must satisfy the equations of balanced momenta. These are,

$$
\begin{aligned}
[N_n] &= \int_{-h/2}^{+h/2} \rho_1 U_1 \; (U_2 - U_1) dz = Pc^2 \; [\bar{u},^o_n] + Rc^2 \; [\psi_{n,n}] \\
[N_{ns}] &= \int_{-h/2}^{+h/2} \rho_1 U_1 \; (\bar{v}_2 - \bar{v}_1) dz = Pc^2 \; [\bar{v},^o_n] + Rc^2 \; [\psi_{s,n}] \\
[M_n] &= \int_{-h/2}^{+h/2} \rho_1 U_1 \; (U_2 - U_1) zdz = Rc^2 \; [\bar{u},^o_n] + Ic^2 \; [\psi_{n,n}] \\
[M_{ns}] &= \int_{-h/2}^{+h/2} \rho_1 U_1 \; (\bar{v}_2 - \bar{v}_1) zdz = Rc^2 \; [\bar{v},^o_n] + Ic^2 \; [\psi_{s,n}] \\
[Q_n] &= \int_{-h/2}^{+h/2} \rho_1 U_1 \; (\bar{w}^o_2 - \bar{w}^o_1) dz = Pc^2 \; [\bar{w},^o_n]
\end{aligned} \tag{11}
$$

In obtaining the above relations, we have retained only the linear terms of the displacements, consistent with the plate theory. In addition, we have assumed density to be a function of z alone in presenting the following quantities

$$(P, R, I) = \int_{-h/2}^{+h/2} \rho_1 \,(1,z,z^2)dz \cong \int_{-h/2}^{+h/2} \rho_o \,(1,z,z^2)dz \qquad (12)$$

where ρ_o is the undisturbed density of the laminae.

Using the relations (7), (8) and (10), we obtain from Equations (11) a system of five linear algebraic equations relating the discontinuities in the normal derivatives of $\bar{u}^o$, $\bar{v}^o$, $\bar{w}^o$, ψ_n and ψ_s

$$\begin{bmatrix} \bar{A}_{11} - Pc^2 & \bar{A}_{16} & \bar{B}_{11} - Rc^2 & \bar{B}_{16} & \bar{A}_{15} \\ \bar{A}_{16} & \bar{A}_{66} - Pc^2 & \bar{B}_{16} & \bar{B}_{66} - Rc^2 & \bar{A}_{56} \\ \bar{B}_{11} - Rc^2 & \bar{B}_{16} & \bar{D}_{11} - Ic^2 & \bar{D}_{16} & \bar{B}_{15} \\ \bar{B}_{16} & \bar{B}_{66} - Rc^2 & \bar{D}_{16} & \bar{D}_{66} - Ic^2 & \bar{B}_{56} \\ \bar{A}_{15} & \bar{A}_{56} & \bar{B}_{15} & \bar{B}_{56} & \bar{A}_{55} - Pc^2 \end{bmatrix} \begin{Bmatrix} [\bar{u},^o_n] \\ [\bar{v},^o_n] \\ [\psi_{n,n}] \\ [\psi_{s,n}] \\ [\bar{w},^o_n] \end{Bmatrix} = 0 \qquad (13)$$

In order for a non-trivial solution to exist, the determinant of the coefficient matrix of Equations (13) must vanish. Thus, five possible wave front speeds, c, may be determined for any given direction $\tilde{n}$.

Since the vanishing determinant represents a fifth order equation in c^2, numerical, rather than analytical, techniques must be used to obtain a solution. However, it is of interest to note that, if the laminated plate has, for each lamina, a monoclinic symmetry (i.e. a plane symmetry with respect to the mid-plane of the lamina) the constants $\bar{A}_{15} = \bar{A}_{56} = \bar{B}_{15} = \bar{B}_{56} = 0$ for all directions in the x,y-plane. In such a case, $[\bar{w},^o_n]$ is uncoupled from the system. Consequently,

$$c_5^2 = \bar{A}_{55}/P \qquad (14)$$

where c_5 represents the propagation speed of the discontinuity $[\bar{w}_{,n}^{o}]$ in the direction $\tilde{n}$.

Furthermore, if, in addition, the plate has a symmetry with respect to the x,y-plane, the bending-extensional couplings $\bar{B}_{ij}$ become zero for all i and j. Then, Equations (13) separate further yielding two quadratic equations whose roots are

$$c_{1,2}^2 = \frac{(\bar{A}_{11} + \bar{A}_{66}) \pm \sqrt{(\bar{A}_{11} - \bar{A}_{66})^2 + 4\bar{A}_{16}^2}}{2P} \tag{15}$$

and

$$c_{3,4}^2 = \frac{(\bar{D}_{11} + \bar{D}_{66}) \pm \sqrt{(\bar{D}_{11} - \bar{D}_{66})^2 + 4\bar{D}_{16}^2}}{2I} \tag{16}$$

In the particular case, such as that considered by Moon [6], the extensional and the bending rigidities are proportional, i.e.,

$$\bar{A}_{ij}/\bar{D}_{ij} = P/I = \text{constant}, \qquad i,j = 1,2,6$$

Equations (14) and (15) are identical causing $c_{1,2}$ and $c_{3,4}$ to coincide (see Equations (17) and (23), Ref. [6]).

III. NUMERICAL ILLUSTRATIONS

For the numerical illustrations, plates which are laminated with unidirectional fiber-reinforced composite layers are considered. The material properties of these layers are described by the following engineering constants*:

$$E_L = 25 \times 10^6 \text{ psi}, \quad E_T = 10^6 \text{ psi}, \quad G_{LT} = 0.5 \times 10^6 \text{ psi}$$

$$\nu_{LT} = 0.25, \quad \nu_{TT} = 0.35, \quad \rho_o = 0.073 \text{ pci}$$

*These values are typical of high modulus graphite-epoxy composites. Such material layers may be considered as being square symmetric. The computation for the plates' rigidities and their transformation to local coordinates was carried out following the outlines in Refs. [2] and [8].

where E is Young's modulus, G is the shear modulus, ν is Poisson's ratio, and the subscripts L and T indicate directions parallel and normal to the fibers, respectively.

We have considered four plates each of which is made of four layers having different lay-up angles and/or lamination sequences, namely: a) (0°/90°/0°/90°), b) (0°/90°/90°/0°), c) (+30°/-30°/+30°/-30°) and d) (+30°/-30°/-30°/+30°), where the angles are positive when measured counterclockwise from the x-axis to the fiber direction.

The roots of the vanishing determinant of Equations (13) are determined by the Newton Raphson technique. In general, five distinct roots representing five possible wave speeds exist in any given direction. However, in certain preferred directions, the material properties are such that repeated roots may exist.

Figures 2 - 5 show the wave velocity surfaces in the first quadrant of the x,y-plane, for (0°/90°/0°/90°), (0°/90°/90°/0°), (30°/-30°/30°/-30°) and (30°/-30°/-30°/30°) laminates, respectively. The velocity is non-dimensionalized by a factor of $(E_T/\rho_0)^{1/2}$. It is pointed out that the slowest velocity surface, which is associated with the transverse displacement $\bar{w}$, is uncoupled from the other four surfaces, since the material is monoclinic. However, all the other four velocity surfaces may be severely coupled. On each of the corresponding wave surfaces, discontinuities in normal forces, shear forces and bending and twisting moments exist simultaneously. Their relative magnitudes may be determined from Equations (13) once the wave speeds, c, have been determined.

Multiple coupled one-dimensional stress waves in a heterogeneous plate were first treated by Wang, Chou and Rose [9] using the method of characteristics. Subsequent experimental investigations in the same problem has not, as yet, confirmed the multiple wave nature associated with laminated plates. Recent experiments conducted at Drexel University using a low frequency ultrasonic transducer to produce normal-to-the-plate pulses showed only two distinct wave groups traveling in the plane of the plate. At present, a definitive conclusion cannot be drawn due to the limited data available.

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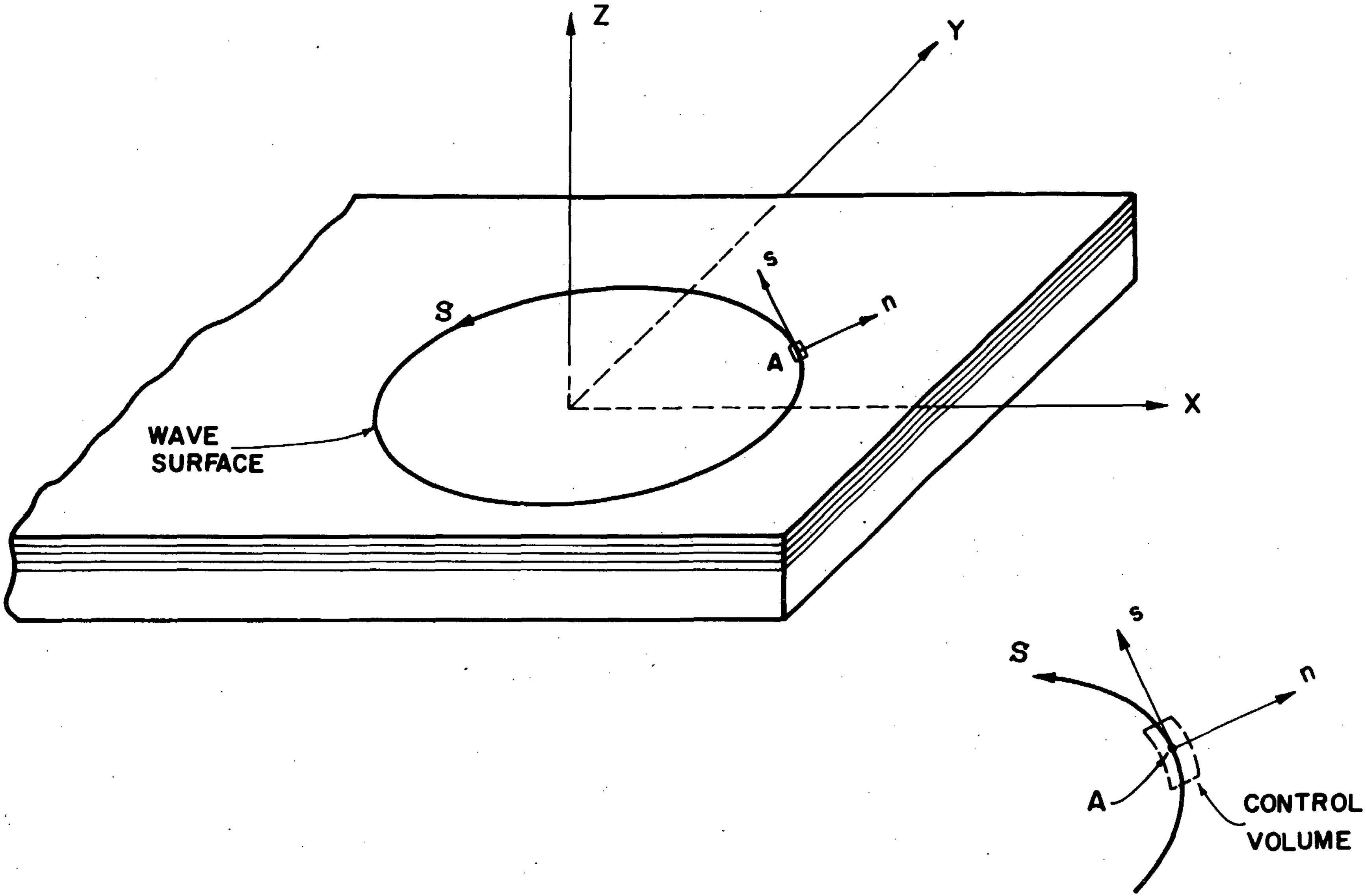

Figure 1. Geometry of the plate and the control volume.

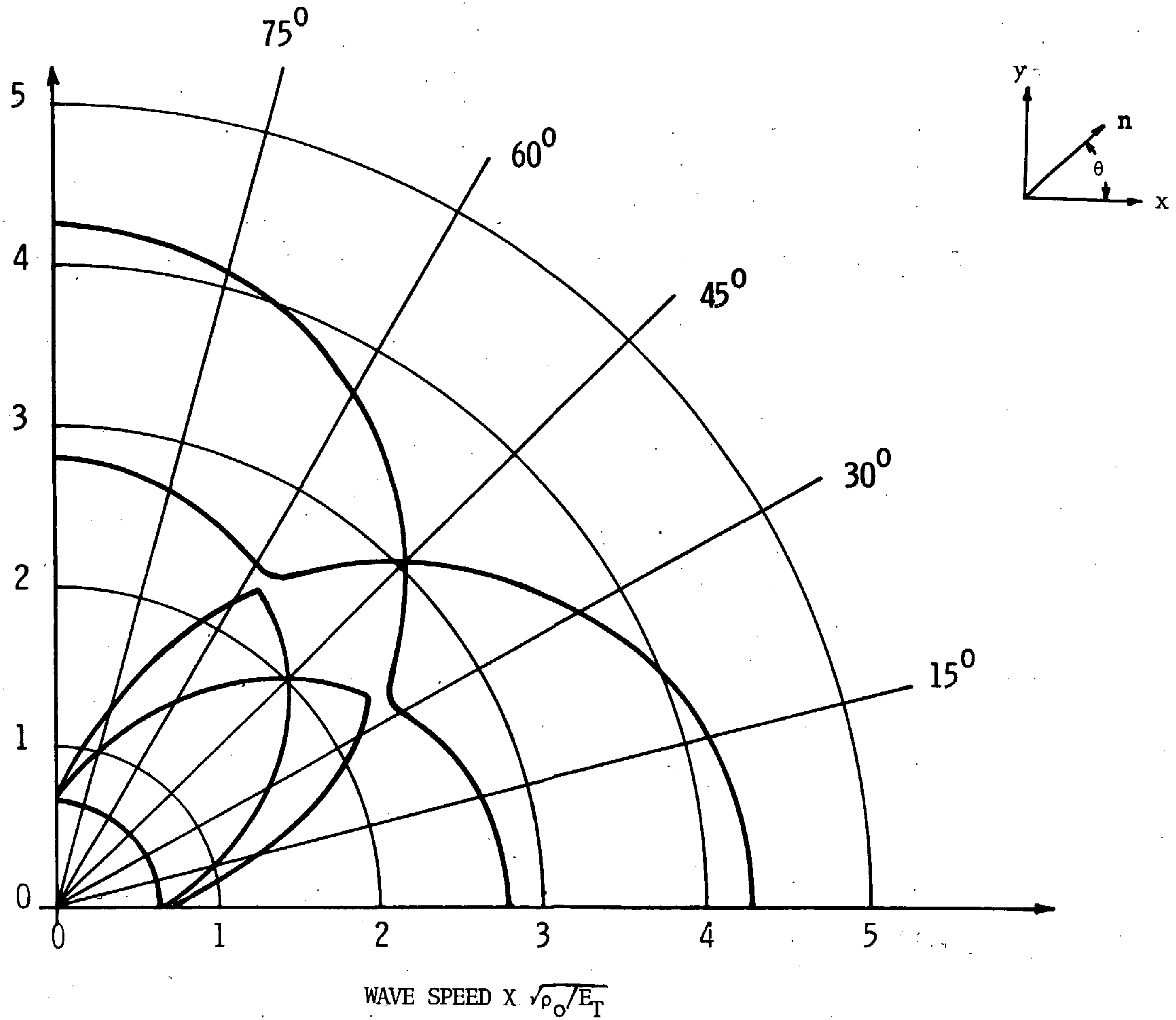

Figure 2. Wave velocity surfaces for the (0°/90°/0°/90°) plate.

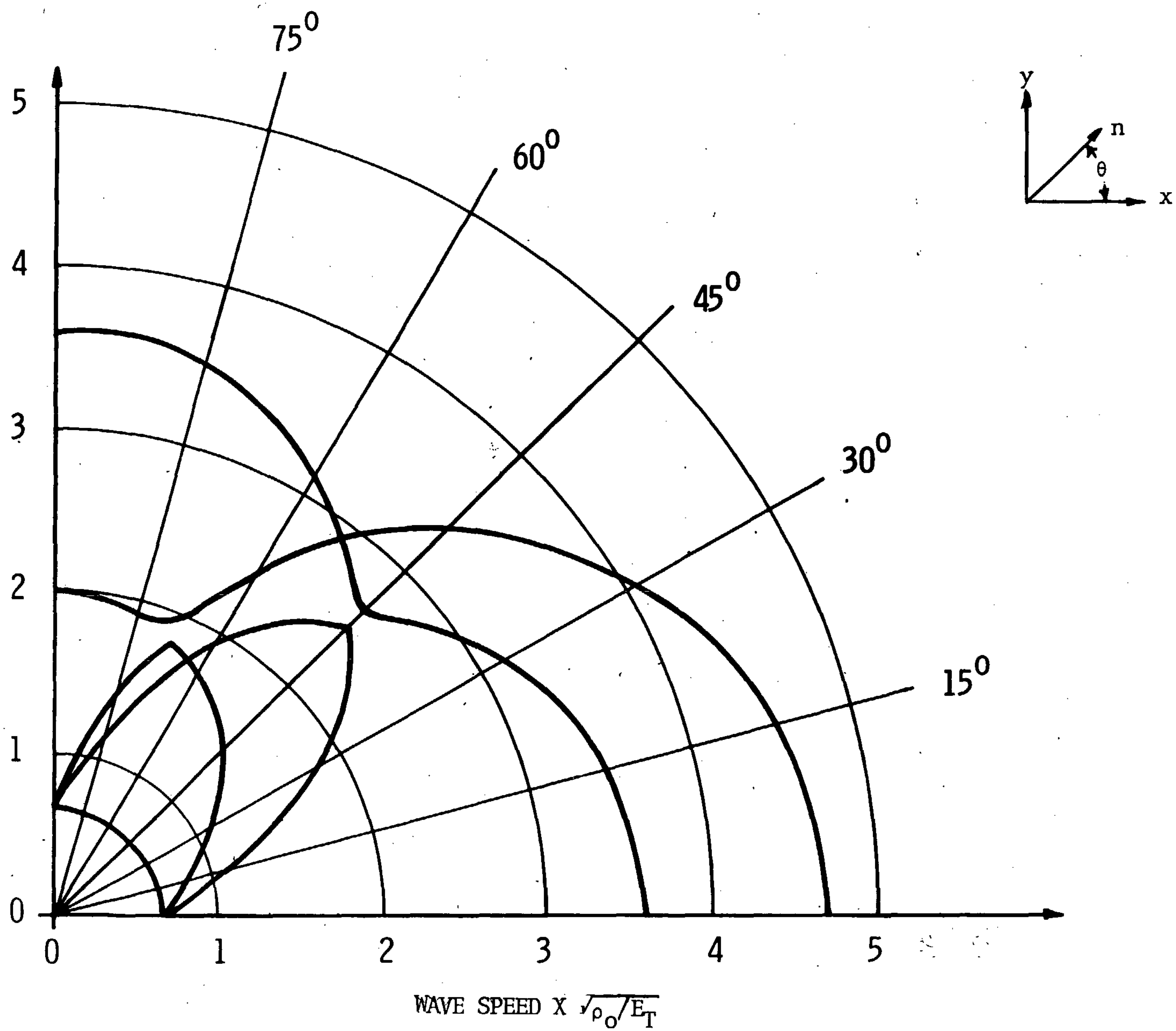

Figure 3. Wave velocity surfaces for the (0°/90°/90°/0°) plate.

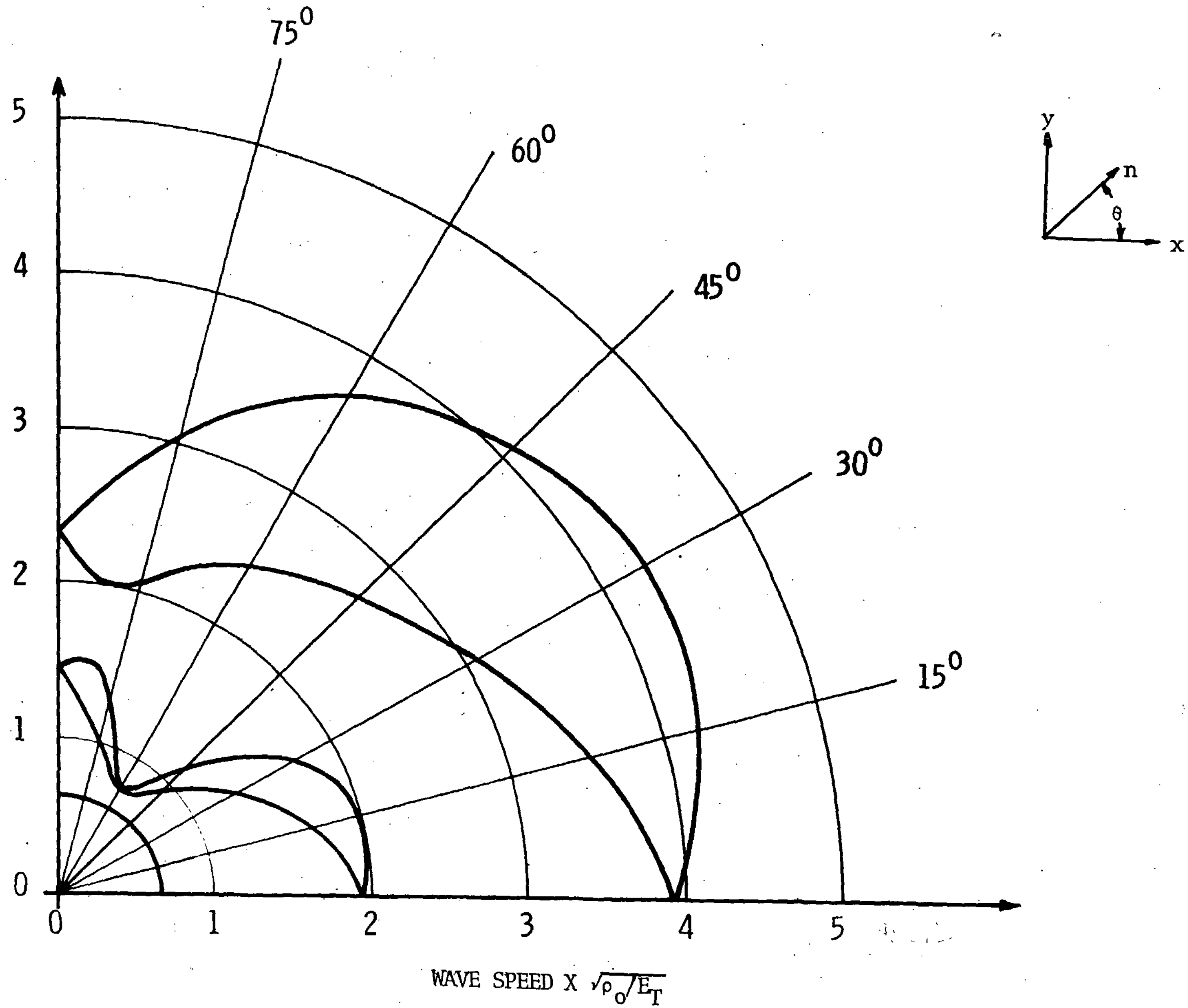

Figure 4. Wave velocity surfaces for the (30°/-30°/30°/-30°) plate.

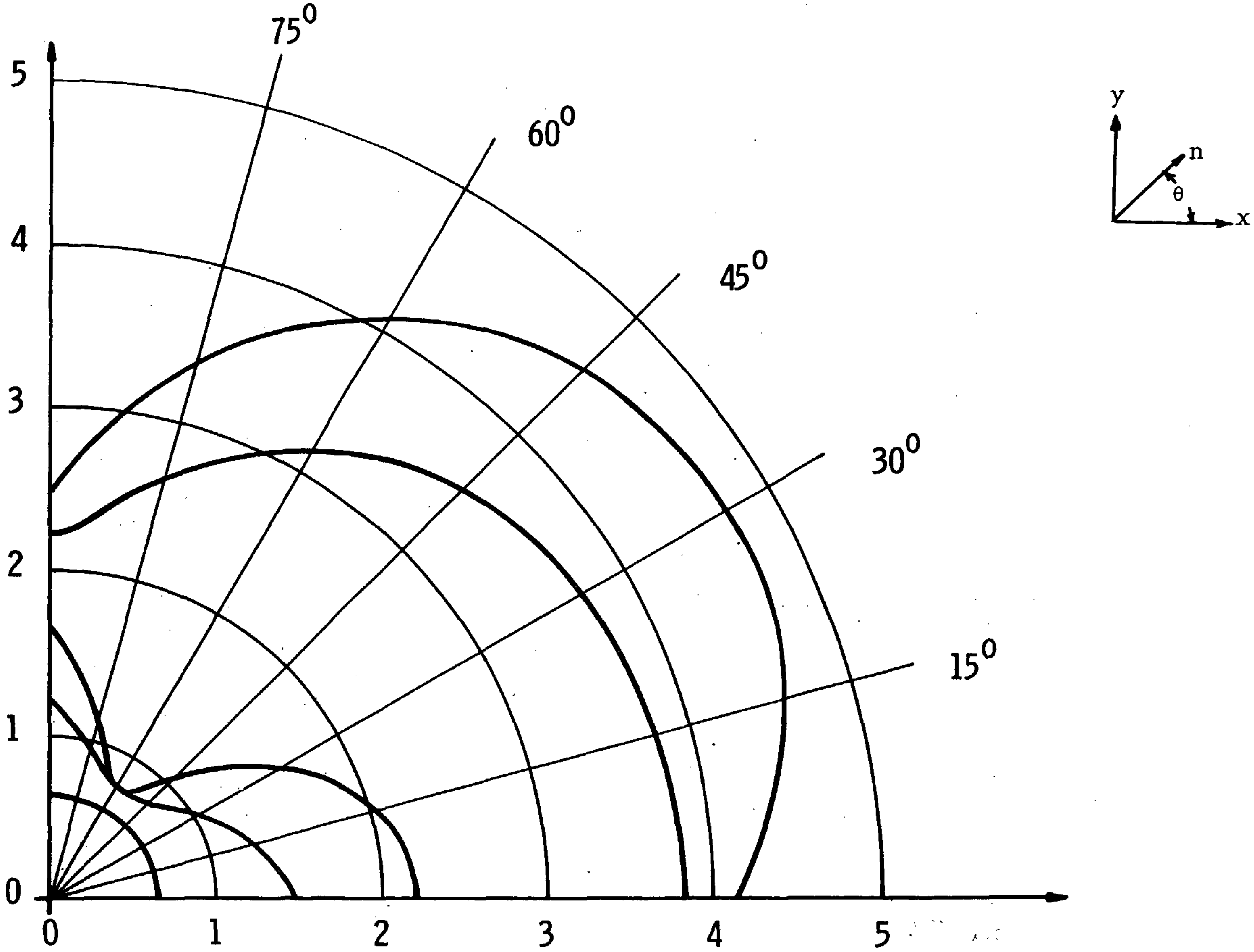

Figure 5, Wave velocity surfaces for the (+30°/-30°/-30°/30°) plate.

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13. ABSTRACT

Elastic stress waves propagating in thin, laminated composite plates are analyzed on the basis of a lamination theory. The theory is based on the Kirchhoff assumptions, but it includes the effects of shear deformation and rotary inertia, similar to Mindlin's theory for homogeneous isotropic plates. The individual layers comprising the plate are assumed to possess different thicknesses and meterial properties. In particular, each layer may be arbitrarily anisotropic. Thus, a general coupling in shear, bending, twisting and extensional effects is present in the plate constitutive relations. This coupling results in simultaneously coupled stress waves propagating in the plane of the plate. Several numerical examples involving laminated fiber-reinforced composite plates are presented.

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